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and so on. Consequently the $n+1$'s horizontal column will be equal to

$$\frac{n}{a+b} = 1 + n + \frac{n \cdot \overline{n-1}}{2} + \frac{n \cdot \overline{n-1} \cdot \overline{n-2}}{2 \cdot 3} + \&c.$$

Which is manifestly equal to the total number of groups contained in the n preceding columns, plus unity.

(To be concluded in our next Number.)

On the Contrivances required to render Contingent Reversionary Interests Marketable Securities. By CHARLES JELlicoe, Esq.,
one of the Vice-Presidents of the Institute of Actuaries, &c.

THIS subject has been so ably treated by Mr. Sang, in the paper transferred by Mr. Thomson's permission to the pages of this Magazine, that it might seem almost superfluous to revert to it. Nevertheless, there are considerations connected with it which I believe to be of some importance, and which that gentleman has not adverted to; and I am therefore induced to submit the following remarks, by way of pendant to his observations:—

To the majority of those I am addressing, no observation will probably appear more trite, than that whenever the idea of an average presents itself, we necessarily connect with it the notion of numbers,—the one qualification being an essential characteristic of the other; so that, in the case of unity, there can of course be no such thing as average. Obvious as is this distinction, it is somewhat remarkable that it has always been, and still is greatly overlooked, in dealing with the values of securities depending upon contingent or uncertain events. That is to say, the values in isolated cases have been confounded with those which are shown to exist when an average really obtains, although the conditions are as essentially different as can well be imagined. Thus the value of an annuity in an isolated case, on a single life, is even at the present time not unfrequently assumed to be identical with that which is found to be the average one amongst many such lives. The same may be said with regard to absolute and contingent reversions; and, as I have observed, with regard to almost all kinds of securities depending on contingent or uncertain events. The origin of so strange an oversight is probably to be found in the circumstance,

that, at an early period, individual dealers in securities of this description were rare, and that the principal buyers of them were either societies, or persons possessing sufficient capital to enter largely into such transactions, and thus to realize the very conditions essential to their doing so with safety; whilst their success induced the smaller capitalist to embark in similar speculations, without observing that the limited extent of his operations gave an entirely different character to them. Of course, the consequences of such a mode of proceeding must occasionally be in the highest degree disastrous; and I cannot help thinking that they have been the cause, to a certain extent, of the little repute in which the craft of an actuary appears sometimes to be held.

From what has been said it will at once be seen, that if a society or capitalist can command the possession of many such securities as those I am speaking of, he can always afford to give a higher price for them than the buyer whose holding will be limited to one or two merely; the latter having to be at some expense to obtain that certainty in his purchase, which the former derives simply from the accident of his having many such; and hence it would seem that these investments have two values, dependent upon the circumstances of the dealers in them. I believe that something of the kind has been found to exist, and would probably still do so, were it not that, from the state of the market, the societies are enabled to command the prices offered for isolated purchases, and thus secure for themselves the difference between the values to which I have alluded.

But, be the actual practice in making purchases of this description what it may, my object is, to insist upon the propriety of regarding them as investments, to be made not subject to any contingency whatever, but as securing to the holder of them, in any case, a given rate of interest so long as he retains them, and as reproducing the capital invested when such interest shall cease to be paid or to accrue. These I believe to be the proper attributes of what are commonly called "marketable securities," for, without such qualification it will generally be found that securities are not ordinarily saleable in the market.

Now, of the various capacities in which the actuary is called upon to exercise his functions, there is none more peculiarly his province than the detecting the causes which may lead to the failure of a security, and the supplying the means which are the most directly adapted to prevent such failure. Some of these means I now propose to discuss, premising, that where contingen-

cies exist, it is obvious we can only get rid of them by a resort to those who make it their business to deal in such contingencies, and who are ready at any time to assume the burden of them at a stipulated price.

The first and most simple of these contingent interests is an annuity on a single life, or a life-interest, as it is sometimes called, whether consisting of a pension or a living, or any annual income whatever depending on the duration of a life. The value of this has been shown by various writers, but first, I believe, by Mr. Griffith Davies, to be truly expressed by the formula

$$\frac{1}{d+p} - 1;$$

where p is the premium for assurance of £1, and d the discount for one year, or the interest upon $1-d$ or v .*

Even to this day, the value of this description of interest is sometimes given as identical with that shown by the ordinary tables of annuities, especially in the case of livings; whereas the purchaser of such a property surely never contemplates the possibility of losing his capital immediately by the death of the person whose life is concerned,—nor is it easy to see why his being a clergyman makes the case any better. This error, however, is wearing out, and I believe every well-educated actuary would now point to the necessity of assurance in such cases, and include the cost of it in his valuation.

The next description of contingent interest I shall cite, is that of the simple reversion. The value of this, I believe, is almost always treated as identical with that of the single premium for an assurance; and the purchaser is consequently exposed to the possibility of the life extending far beyond the average term, and to the contemplated rate of interest being reduced to one which would have put the purchase out of the question, had the buyer understood that such was likely to be the case. It is true, that the reverse may happen, and he may be as great a gainer on the other hand; but, as I have said, he ought not to be placed in this situation,—that is, the *price he should pay* ought to be such as would

* The truth of the several formulæ given in this paper will be obvious on inspection, by bearing this in mind:—Thus, since the purchaser must always have interest on his capital for the year in which the death happens, the advance for £1 must not exceed v or $1-d$; in this case, too, he must deduct p , the first year's premium; and since d is the interest upon $1-d$, the annuity he should get for $1-d-p$ is $(d+p)$; so that £1 annuity is worth $\frac{1-(d+p)}{d+p}$ or $\frac{1}{d+p} - 1$; that is to say, the sum to be assured, less the first year's premium and the last year's interest.

enable him to get rid of the contingencies if he wished it, whether he thinks proper actually to do so or not, without losing any of the advantages naturally stipulated for in such bargains. Hence the true value of this kind of investment should be expressed by the formula

$$1 - d(1 + A).^*$$

It being necessary in this case to resort to an annuity instead of an Assurance Society, and by its means secure interest on the capital, in the interval between the outlay of it and the realization of the reversion, be that interval what it may.

From what has been said as regards these two cases, it will readily be concluded, that I consider the ordinary method of estimating the life-interest and the reversion in an estate to be erroneous. I believe this last is always taken as equal to $\frac{1}{r} - A$, a mode of estimation in isolated cases, the inaccuracy of which, I have no doubt, many purchasers have found to their cost. The life-interest and the reversion, I would contend, should be determined in the manner now pointed out; and their values, so stated and explained, would satisfy the conditions naturally bargained for by those who may become purchasers of them.

In the case of a contingent reversionary sum,—that is to say, of a sum payable at death of A, if B be then living,—the purchaser has to guard against the premature death of the latter, and to provide interest on his capital whilst both are living. For the first object, he must apply to an Assurance Office; and for the second, to an Annuity Company;—and the cost of these will have to be taken into consideration in the price to be paid for the interest in question. Hence the value of it will be truly denoted by the formula

$$1 - (d + p)(1 + AB); \dagger$$

p being here the annual premium for assuring the life of B against

* Here again the advance for £1 reversion is v or $1 - d$, in addition to which, the interest upon $1 - d$ during the life of A has to be provided for. The sum to be given is therefore $1 - d - dA$, or $1 - d(1 + A)$, or $v - dA$.

† Here the purchaser, for the same reason as before, must limit his advance for £1 to v ; from which he has to deduct the first year's premium p , and the cost of an annuity to provide interest and premium—that is to say, of $(d + p)$ —during the joint-lives. Hence the difference will be $1 - d - p - (d + p)AB$, or $1 - (d + p)(1 + AB)$. He will thus receive interest, and have the means of paying the premium, while both lives are in being; and should A die first, he will get £1 = $v + d$, the total outlay, with interest upon it for the last year, from the estate; or should B die first, he will get it from the Assurance Office.

that of A, and AB the value or cost of an annuity of £1 during the joint-lives of A and B.

The description of contingent interest which I shall last advert to, in this paper, is that called a contingent reversionary annuity, —that is to say, an annuity on the life of B, to commence at decease of A. In this case the desiderata are much the same as in the foregoing; save that the contingencies do not terminate with the dissolution of the joint-existence of the two lives, and that it becomes necessary to provide against the decease of B after he shall have survived A, as well as during the lifetime of A. This can only be done by effecting at once (for it would not be safe to defer it) an absolute assurance on the life of B; and hence, if we now call p the annual premium for such an assurance, the formula for the value of this description of security will be

$$\frac{1}{p+d} - (1+AB).*$$

These several methods, it will be observed, are perfectly analogous. They stipulate in each case for the ultimate recovery of the capital advanced, and for interest in the interval; and although the values deduced by these means may appear to be small, they cannot be considered as unreasonably so, whilst no more than this is demanded. It will be of course for the seller to see that the rates at which the assurance and annuity are taken are not excessive, and for the buyer to look to their being not only actually practicable ones, but such as can be acted upon by an Office with perfect safety. As I have already hinted, it does not follow that the arrangements should be actually carried out. What the actuary has to provide for is that their being carried out shall not be inconsistent with the realization of such advantage, in the shape of interest, as the persons consulting him may have stipulated for, or may be desirous of obtaining. Having placed his client in this

* In this case, as in the foregoing ones, the advance must be limited to $1-d$; from which have to be deducted the first year's premium, and the cost of an annuity equivalent to the annual premium and interest, whilst both A and B are living. The sum to be given therefore for a contingent reversionary annuity of $p+d$, under these circumstances, would be $1-d-p-(p+d)AB$, or $1-(p+d)(1+AB)$. The purchaser would then receive his annuity of $(p+d)$ so long as B is alive,—viz., from the Annuity Company whilst A is living, and from the estate should he die; and at B's death he would recover his capital as before, with interest for the year in which his death occurred. But if $p+d$ annuity is worth $1-(p+d)(1+AB)$, £1 annuity is worth $\frac{1-(p+d)(1+AB)}{p+d}$, or $\frac{1}{p+d} - (1+AB)$, as above given. In this expression, $\frac{1}{p+d}$ represents the sum to be assured; and in strict analogy with the former cases, is identical with that to be given for such an annuity, after deducting the first year's premium, the last year's interest, and the sum required to provide both so long as the joint-lives are in being.

position, it will be for the latter to determine how much of the risk he will take upon himself, or how much buy off; but it is of no less importance to the credit of the one, than it is to the interest of the other, that a clear understanding should be had of what the risks or contingencies are, and how and at what cost they are to be provided against.

The rate at which d is to be taken—that is to say, the rate of interest to be realized by investments of this description—will necessarily vary with the degree of the sufficiency and validity of the particular security. Where these are unobjectionable, and the proper allowance made for the contingencies I have been discussing, there does not seem any reason why the legal rate of interest should be exceeded. It is true that extra risks may be incurred by the vendor or mortgagor going abroad; and should an annuity be actually purchased, difficulty may arise from the same cause in the receipt of it. But it will be for the purchaser to take due care in every case that a proper margin is left, not only to provide for any extra payment which may be required, but sufficient to give the vendor a permanent interest in the reversionary estate. With such an interest, it would not often happen that the latter would place himself out of reach, or resort to a locality out of the pale of assurance; nor would it, under the circumstances of the present time, be easy for him to do the latter, were he so disposed.

From what has been said, it will be readily perceived that the redemption-money in every case should be the total amount originally advanced, or supposed to be advanced, viz. v or $1-d$; in addition to which, interest upon it for half-a-year might be fairly demanded,—it being reasonable to expect that so much time would elapse before a similar investment could be made. But, unless any stipulation be made to the contrary, it would appear to be no more than just that the policy of assurance, and the deed of annuity, or at least the then value of them, be handed over to the vendor, should the redemption take place whilst they are both in force; the other party having derived all the benefit from them which he could reasonably look for. I am aware that the practice has been generally different from this, and that the legal dictum is precisely the reverse of it; but it is scarcely justifiable on any grounds, and is, I believe, now much on the decline. It is better that all these arrangements should be adjusted with a strict regard to equity; and that, if a hard bargain be to be made, it should consist and appear, as it ought, in an increased rate of interest.

As it may be satisfactory to see something of the mode in

which these methods operate, I will conclude this paper by an example such as very commonly occurs in practice, and exhibit in figures the results which one of them affords. We will suppose that B, aged 30, is entitled to a considerable life estate at the death of A, aged 70, and requires to know to what extent he must charge it for the advance of £5000; also, what the redemption-money will be before the decease of A, and what after such decease.

The formula to be used in this case is

$$\frac{1}{p+d} - (1 + AB);$$

and if we take $p = .0215$ * a moderate non-participating rate, $d = .0476$ and $AB = 6.472$,* (Carlisle mortality, and $3\frac{1}{2}$ per cent. interest), we find the value of a contingent reversionary annuity of £1 under these circumstances to be 6.9956. The annuity to be charged for £5000 is therefore £714.75.

Now the sum to be assured in this case will be $\frac{714.75}{p+d} =$ £10340; and the annual premium will therefore amount to $.0215 \times 10340$, or £222.31.

The interest will also be $714.75 - 222.31$, or £492.44; and the sum to be expended to provide the annuity during the joint-lives will be 6.472×714.75 , or £4625.86.

Hence the total amount laid out, or supposed to be laid out, will consist of—

	£
The sum advanced . . .	5000.00
The first year's premium . . .	222.31
AND	
The cost of the annuity . . .	4625.86
That is to say . . .	<u>£9848.17</u>

the interest upon which being £492.44, as above stated, very nearly, proves the accuracy of the work.

Thus, it will be seen that the lender will get his annuity of £714.75 from the Office while the two lives are in being, and from the estate when the elder one fails, recovering his capital and interest (viz. £10340) by means of the assurance at the decease of the younger one.

If the annuity be redeemed before the death of A, the redemp-

* These are what may be called *practicable* rates. To assume ones less costly, or much less costly, would be to build on a false foundation!

tion-money should be equal to the total sum laid out, with half a year's interest upon it,—less the value, at the time, of the annuity on the joint-lives, and the assurance.

After the death of A, the value of the assurance only, at the time of redemption, would have to be deducted,—the annuity on the joint-lives being of course at an end.

*On the Extra Premiums charged for Assurance of the Lives of
Persons going Abroad.*

THE Council of the Institute of Actuaries are making a laudable attempt to collect such data as the Assurance Offices possess, with a view to determine the mean duration of human life in different parts of the Globe; and thence to ascertain the proper rate of premium to be charged in the several localities to which persons whose lives are assured may be desirous of proceeding. The almost entire absence of any authentic information upon this subject renders such a proceeding of the greatest importance and interest; since it is only by such means we can hope to be able to substitute, for the present uncertain and arbitrary charges, rates which are based upon actual experience, and which are justified by the circumstances of each particular case. It is hardly to be expected, however, that anything more than a general approximation to the truth can be obtained as yet in this way. If the range of country in which any observations are made be much restricted, the numbers observed upon must be necessarily limited; whilst to include a sufficiently considerable body, a variety of climates must be grouped together. Nevertheless, the inquiry can hardly fail to be attended with considerable advantage.

Pending its completion, it will perhaps be acceptable to some of our readers to have before them, in a compendious form, the rates adopted some time back by the managers of the Scotch Offices, and acted upon pretty generally by them, we believe, at the present time. We are much inclined to agree with those gentlemen in preferring the old method of making an equal annual addition to the European-rate, instead of adopting a new table for any given climate, at least until the rate of mortality for such climate is satisfactorily determined. In practice it is much more convenient, especially as regards valuations, and the changes consequent upon the lives passing from one climate to another;